# Disentangling Reputational Effects in Alliances

## E-Companion: Proofs and Supplementary Material

### EC.1 Prior Notes

#### EC.1.1 Notation and Equilibria

Throughout the analytical proofs, we denote a strategy of the two firms by the function $x(q)$: $[0,1] \times [0,1] \to \{0,1\}$. For each profile of firms' qualities $q = (q_A, q_B)$, $x(q)$ specifies the firms' decision of whether to form an alliance, where $x(q) = 0$ if firms choose not to form an alliance and thus to implement their projects independently (i.e., $d = I$), and $x(q) = 1$ if firms choose to form an alliance and thus to implement some of their projects jointly (i.e., $d = J$).

Moreover, our analysis focuses on *sabotage-free* equilibria, that is, equilibria where firms' joint profits are never higher if project $P_C$ fails than if it succeeds and, therefore, firms never have the incentive to make project $P_C$ fail on purpose.

#### EC.1.2 Belief Updating by Consumers

We assume that consumers may update their beliefs at the interim stage, but *only* when they observe an alliance. This assumption is consistent with situations where an alliance is not anticipated by consumers. So, if consumers observe that the *status quo* is maintained, they will not make any inferences about firms' qualities. One way of formalizing this idea is by assuming that there is a continuum of firms in the economy and only a countable (i.e., with probability measure zero) subset of them face the opportunity to form an alliance. In this case, if consumers do not observe which firms face an opportunity to form an alliance, they will not update their beliefs about the quality of any two firms that do not form an alliance.

However, as is the case in our model, if consumers observe that two firms formed an alliance, they may update their beliefs about the firms' qualities. When updating their beliefs, consumers take into account their initial beliefs about the firms' qualities, as well as the firms' strategy $x(q)$ in equilibrium. Note that we mentioned that consumers *may* update their beliefs if they observe an alliance. To be more precise, consumers always go through the process of revising their beliefs when they observe an alliance. Nonetheless, it is possible that, at the end of the revision process, their interim beliefs are identical to their initial beliefs. Next, we detail precisely how consumers' revise their beliefs.

The notion of Bayesian equilibrium implicitly assumes that consumers know the firms' strategy $x(q)$ in equilibrium. That is, consumers' expectations about how firms behave when they face the opportunity to form an alliance are correct. A Bayesian equilibrium also requires that consumers use Bayes' rule to update their beliefs. Thus, when consumers observe an alliance at the end of period zero, their interim beliefs are obtained by applying Bayes' rule to their initial beliefs $G(q)$ (with corresponding density $g(q)$) in the following way: $g_1^J(q) = x(q)g(q)/\int x(\hat{q})dG(\hat{q})$. A technical assumption that we make is that the distribution $G(q)$ is atomless and the corresponding density $g(q) > 0$, for all firm qualities $q = (q_A, q_B) \in [0,1] \times [0,1]$. Note that consumers may have different initial beliefs about qualities $q_A$ and $q_B$. Furthermore, following the standard practice in models using the Bayesian equilibrium as a solution concept, we assume that consumers' initial beliefs (or priors) are correct—that is, $G(q)$ is also the distribution from which firms' qualities are drawn.

Consumers update their beliefs also at the ex-post stage after observing the performance of project $P_C$ at the end of period one. After observing the performance of project $P_C$, consumers' ex-post beliefs are obtained by applying Bayes' rule to their interim beliefs $G_1^d(q)$ (and $g_1^d(q)$) in the following way: $g_2^d(q|\varphi) = \Pr[\varphi|q, d]g_1^d(q)/\int \Pr[\varphi|\hat{q}, d]\, dG_1^d(\hat{q})$, where $\Pr[\varphi = s|q, d]$ and $\Pr[\varphi = f|q, d]$ represent, respectively, the probability that project $P_C$ succeeds and the probability that project $P_C$ fails, both conditional on firms' qualities $q = (q_A, q_B)$ and decision $d \in \{I, J\}$. Note that $\Pr[\varphi = s|q, d = J] = q_{P_C}^J = \alpha_A^C q_A + \alpha_B^C q_B$, $\Pr[\varphi = s|q, d = I] = q_{P_C}^I = q_A$, and $\Pr[\varphi = f|q, d] = 1 - \Pr[\varphi = s|q, d]$.

#### EC.1.3 Quality of Jointly Implemented Projects

The quality of a jointly implemented project $P_j$ (for $j \in \{A, B, C\}$) is given by $\alpha_A^j q_A + \alpha_B^j q_B$, the weighted average of the qualities of the two firms, where the weights are the firms' participation levels in the project. This is a simple and analytically tractable way of capturing the fact that firms' individual qualities have spillover effects on jointly implemented projects. Note that the results obtained in this paper are not driven by this particular specification of how firms' qualities affect the quality of joint projects. More specifically, the results remain valid if the following fairly innocuous properties hold: (i) the quality of a joint project is strictly between the qualities of the participating firms whenever those qualities are different; (ii) the marginal impact of a firm's quality on the quality of a joint project in which it participates is strictly positive; (iii) this marginal impact is increasing in the firm's participation level in the joint project and goes to zero as the firm's participation level goes to zero; and (iv) the marginal impact is bounded away from zero whenever the firm's participation level in the joint project is strictly positive.

### EC.1.4 Dominant Reputation-Independent Synergies

We derive here a necessary and sufficient condition that ensures that reputation-independent synergies *dominate* reputational considerations in firms' decision and, as a result, that firms always choose an alliance (i.e., $x(q) = 1$ for all $q \in [0,1] \times [0,1]$) regardless of reputational considerations. To derive that condition, we need to compare firms' expected joint profits for extreme cases under independent project implementation and under an alliance.

Let us start with firms' expected joint profits under independent project implementation, which are given by $\Pi^I(q_A) = V_A + V_B + 2V_C - K + \mu\{r_A V_C + [q_A r_A^I(s) + (1 - q_A) r_A^I(f)](V_A + V_C) + r_B V_B\}$. These profits are the highest possible when the quality of firm $A$ $q_A$ is one (since by Lemma EC.1 presented below, $r_A^I(s) > r_A^I(f)$), and are then given by $\Pi^I(q_A = 1) = V_A + V_B + 2V_C - K + \mu\{r_A V_C + r_A^I(s)(V_A + V_C) + r_B V_B\}$.

Let us now consider firms' expected joint profits under an alliance, which are given by $\Pi^J(q_A, q_B) = V_A + V_B + 2V_C - K + S + \mu\left\{r_{P_C}^J V_C + \sum_{j\in\{A,B,C\}} r_{P_j}^J(f) V_j + q_{P_C}^J \times \sum_{j\in\{A,B,C\}}[r_{P_j}^J(s) - r_{P_j}^J(f)]V_j\right\}$. These profits are the lowest possible if we assume that firms' reputations drop to zero when consumers observe an alliance—that is, if we assume that, in the event of an alliance, firms' interim and ex-post reputations become zero. In that case, firms' expected joint profits are given by $\underline{\Pi^J} = V_A + V_B + 2V_C - K + S$.

Hence, firms always form an alliance in equilibrium (i.e., $x(q) = 1$ for all $q \in [0,1] \times [0,1]$) regardless of reputational considerations if the following condition is satisfied $\underline{\Pi^J} > \Pi^I(q_A = 1) \Leftrightarrow S > \mu[r_A V_C + r_B V_B + r_A^I(s)(V_A + V_C)]$. If this condition is satisfied, then reputation-independent synergies $S$ are dominant and reputational considerations have no impact on firms' decision. If this condition is not satisfied, reputational considerations will affect firms' decision.

Finally, from result (i) of Lemma EC.1 (presented below) we have $r_A^I(s) = r_A + \frac{\sigma_A}{r_A}$, so we can transform the above condition into $S > \mu\left[r_A V_C + r_B V_B + (r_A + \frac{\sigma_A}{r_A})(V_A + V_C)\right] = \mu\left[2r_A V_C + r_A V_A + r_B V_B + \frac{\sigma_A}{r_A}(V_A + V_C)\right]$, so that it only depends on exogenous model parameters.

### EC.1.5 Lemma EC.1

In this e-companion, we prove a technical lemma—Lemma EC.1—that is not fully reported in the body of the paper but that is used to obtain our results. Lemma EC.1 uses consumers' ex-post beliefs ($G_2^d(q|\varphi)$ and $g_2^d(q|\varphi)$) to compare firms' interim and ex-post reputations. It shows that the impact of the performance of an independently implemented project (in our model, project $P_C$) on the reputation of a given firm (in our model, firm $A$) is always positive. It also shows that the impact of a jointly implemented project on the reputation of a given firm depends on consumers' perceived variance of the firm's quality ($\sigma_i^J$ for all $i \in \{A, B\}$), as well as on consumers' perceived correlation (more specifically, covariance) between that firm's quality and the quality of the other firm ($\sigma_{A,B}^J$).

LEMMA EC.1.

*(i) Under independent implementation of a project (project $P_C$), the ex-post (i.e., period two) reputations of firm A in case of a success ($\varphi = s$) and of a failure ($\varphi = f$) are:*

$$r_A^I(s) = r_A + \frac{\sigma_A}{r_A} \text{ and } r_A^I(f) = r_A - \frac{\sigma_A}{1-r_A},$$

$$\text{such that } r_A^I(s) - r_A^I(f) = \frac{\sigma_A}{(r_A)(1-r_A)},$$

*where $\sigma_A$ is the variance of $q_A$, according to initial beliefs $G(q)$ (which are equal to interim beliefs $G_1^I(q)$).The ex-post reputation of firm B is the same as the initial (and interim) reputation ($r_B = r_B^I = r_B^I(s) = r_B^I(f)$).*

*(ii) Under joint implementation of a project (project $P_C$), the ex-post reputations of firm $i \in \{A, B\}$ in case of a success ($\varphi = s$) and of a failure ($\varphi = f$) are:*

$$r_i^J(s) = r_i^J + \frac{\alpha_i^C \sigma_i^J + (1-\alpha_i^C)\sigma_{A,B}^J}{\alpha_A^C r_A^J + \alpha_B^C r_B^J} \text{ and } r_i^J(f) = r_i^J - \frac{\alpha_i^C \sigma_i^J + (1-\alpha_i^C)\sigma_{A,B}^J}{1-\alpha_A^C r_A^J - \alpha_B^C r_B^J},$$

$$\textit{such that } r_i^J(s) - r_i^J(f) = \frac{\alpha_i^C \sigma_i^J + (1-\alpha_i^C)\sigma_{A,B}^J}{(\alpha_A^C r_A^J + \alpha_B^C r_B^J)(1-\alpha_A^C r_A^J - \alpha_B^C r_B^J)},$$

*where $\sigma_i^J$ and $\sigma_{A,B}^J$ are, respectively, the variance of $q_i$ and the covariance of $q_A$ and $q_B$, according to interim beliefs $G_1^J(q)$.*

*Proof of Lemma EC.1.* We start with result (ii). By definition, $r_i^J(\varphi) = \int q_i dG_2^J(q|\varphi)$ for all $i \in \{A, B\}$ and $\varphi \in \{f, s\}$. Applying Bayes' rule to interim beliefs, we obtain that $g_2^J(q|\varphi) = \Pr[\varphi|q] g_1^J(q) / \int \Pr[\varphi|\hat{q}]\, dG_1^J(\hat{q})$, where $\Pr[s|q] = \alpha_A^C q_A + \alpha_B^C q_B$ and $\Pr[f|q] = 1 - \alpha_A^C q_A - \alpha_B^C q_B$. Using this expression for $g_2^J(q|\varphi)$ and applying standard integration properties, we obtain $r_i^J(\varphi) = \int q_i \Pr[\varphi|q]\, dG_1^J(q) / \int \Pr[\varphi|\hat{q}]\, dG_1^J(\hat{q})$ for all $i \in \{A, B\}$. The first two equations in result (ii) are obtained by using this equation when $\varphi = s$ and $\varphi = f$ and doing the following for the two cases: (i) replacing $\Pr[s|q]$ with $\alpha_A^C q_A + \alpha_B^C q_B$ and $\Pr[f|q]$ with $1 - \alpha_A^C q_A - \alpha_B^C q_B$; (ii) using the fact that $var(x) = E(x^2) - [E(x)]^2$ and $cov(x, y) = E(xy) - E(x)E(y)$ for any given random variables $x$ and $y$; (iii) rearranging the terms; (iv) noticing that by the definition of interim reputation $\int \alpha_i^C q_i dG_1^J(q) = \alpha_i^C \int q_i dG_1^J(q) = \alpha_i^C r_i^J$. The last equation in result (ii) follows directly from the first two equations in the same result.

We now prove result (i). $r_A^I(s)$ and $r_A^I(f)$ follow directly from result (ii) by noting that $r_A^I(\varphi) = r_A^J(\varphi)$ for all $\varphi \in \{f, s\}$ when $\alpha_A^C = 1$ and $G_1^I(q) = G(q)$. Similarly, $r_B^I(s)$ and $r_B^I(f)$ follow directly from result (ii) by noting that $r_B^I(\varphi) = r_B^J(\varphi)$ for all $\varphi \in \{f, s\}$ when $\alpha_B^C = 0$ and $G_1^I(q) = G(q)$, and that, under initial beliefs $G(q)$, qualities $q_A$ and $q_B$ are independent (meaning that their covariance is zero). ■

## EC.2 Proofs of the Propositions and Lemmas in the Body of the Paper

*Proof of Lemma 1.* The proof of Lemma 1 follows directly from the above prior note to the proofs EC.1.4 on dominant reputation-independent synergies. ■

*Proof of Lemma 2.* Let us start with the proof of result (i), the case of an equilibrium configuration in which $x(q) = 1$ for all $q \in [0,1] \times [0,1]$. In any equilibrium configuration in which firms form an alliance for at least some combination of their qualities, consumers' interim beliefs in the event of an alliance are obtained by applying Bayes' rule to their initial beliefs $G(q)$. Thus, $g_1^J(q) = x(q)g(q) / \int x(\hat{q}) dG(\hat{q})$. It follows that in an equilibrium where $x(q) = 1$ for all $q \in [0,1] \times [0,1]$ we have $G_1^J = G$. Thus, in this type of equilibrium $G_1^J = G_1^I = G$. Given $G$ and the other basic parameters of the model, such an equilibrium configuration exists if and only if $\Pi^J(q) > \Pi^I(q)$ for all $q \in [0,1] \times [0,1]$.

We conclude the proof of result (i) by providing an example where indeed $\Pi^J(q) > \Pi^I(q)$ for all $q \in [0,1] \times [0,1]$. Suppose that initial beliefs about firm $i$'s quality are described by the density function $f_i(q_i) = [q_i^{\omega_i - 1}(1 - q_i)^{\beta_i - 1}] / \int \hat{q}_i^{\omega_i - 1}(1 - \hat{q}_i)^{\beta_i - 1} d\hat{q}_i$ for all $i \in \{A, B\}$. Thus, $f_i(q_i)$ is a Beta distribution. Hence, $g(q) = f_A(q_A) \times f_B(q_B)$. Let $\omega_A = 46.625$, $\beta_A = 139.88$, $\omega_B = 13.313$, and $\beta_B = 4.4375$. Let also $V_A = V_C = 2$, $V_B = 1$, $\alpha_i^j = 0.5$ for all $i \in \{A, B\}$ and $j \in \{A, B, C\}$, $K = 1$, and $S = 0$. Given the above initial beliefs, $r_A = 0.25$ and $r_B = 0.75$. Since $G_1^I = G_1^J = G$, interim reputations satisfy $r_i^I = r_i^J = r_i$ for all $i \in \{A, B\}$. Applying Bayes' rule to interim beliefs we obtain ex-post beliefs. Hence, $g_2^d(q|\varphi) = \Pr[\varphi|q, d] g_1^d(q) / \int \Pr[\varphi|\hat{q}, d]\, dG_1^d(\hat{q})$ and since in this type of equilibrium $G_1^I = G_1^J = G$, we can easily obtain ex-post beliefs $G_2^I$ and $G_2^J$. Using those ex-post beliefs, we can then obtain the ex-post reputations $r_A^I(f) = 0.24867$, $r_A^I(s) = 0.254$, $r_B^I(f) = r_B^I(s) = r_B = 0.75$, $r_A^J(f) = 0.249$, $r_A^J(s) = 0.251$, $r_B^J(f) = 0.74$, and $r_B^J(s) = 0.76$. Using these reputations, we can obtain $\Pi^J(q)$ and $\Pi^I(q)$. In this case, we have $\partial(\Pi^J(q) - \Pi^I(q))/\partial q_A = 0.0117 > 0$ and $\partial(\Pi^J(q) - \Pi^I(q))/\partial q_B = 0.330 > 0$. Thus, we will have $\Pi^J(q) > \Pi^I(q)$ for all $q \in [0,1] \times [0,1]$ if $\Pi^J(0,0) > \Pi^I(0,0)$, which is the case since $\Pi^J(0,0) = 6 + \mu 3.4725$ and $\Pi^I(0,0) = 6 + \mu 2.2447$ (recall that by assumption $\mu > 0$).

Let us now turn to the proof of result (ii) of Lemma 2. We show that for some parameter values an equilibrium configuration in which $x(q) = 0$ for all $q \in [0,1] \times [0,1]$ exists. In this type of equilibrium configuration, an alliance occurs with probability zero, that is, it is off the equilibrium path. The notion of Bayesian equilibrium imposes no restrictions on beliefs off the equilibrium path, which means that one can choose any possible interim and ex-post beliefs $G_1^J$ and $G_2^J$ off the equilibrium path. In particular, since $g(q) > 0$ for all $q \in [0,1] \times [0,1]$, we can choose $G_1^J$ and $G_2^J$ such that $r_i^J$ and $r_i^J(\varphi)$ (for all $i \in \{A, B\}$ and $\varphi \in \{f, s\}$) are as small as desired. So, we can choose them so that $r_i^J = r_i^J(\varphi) = 0$ (for all $i \in \{A,B\}$ and $\varphi \in \{f,s\}$), in which case $\Pi^J = V_A + V_B + 2V_C - K + S$. We need to compare these joint profits with firms' joint profits under independent project implementation. If firms implement their projects independently, their joint profits are given by $\Pi^I(q) = V_A + V_B + 2V_C - K + \mu\{r_A V_C + [q_A r_A^I(s) + (1-q_A) r_A^I(f)](V_C + V_A) + r_B V_B\}$. These joint profits depend only on firm $A$'s quality $q_A$ and are increasing in it (note that from result (i) of Lemma EC.1 it follows that $r_A^I(s) > r_A^I(f)$). Thus, an equilibrium in which firms always choose to implement their projects independently regardless of their qualities exists if and only if $\Pi^J \le \Pi^I(q_A = 0, q_B)$, which is equivalent to $S \le \mu\{r_A V_C + r_A^I(f)(V_C + V_A) + r_B V_B\}$. This condition is satisfied for example when $S \le \mu\{r_A V_C + r_B V_B\}$.

Finally, we prove result (iii) of Lemma 2. Consider an arbitrary equilibrium configuration in which, for some set $H \subset [0,1] \times [0,1]$, we have $x(q) = 1$ if $q \in H$ and $x(q) = 0$ if $q \notin H$. By the definition of Bayesian equilibrium, firms optimize given interim and ex-post beliefs (and their corresponding reputations), forming an alliance if and only if their qualities $q$ are such that $\Pi^J(q) > \Pi^I(q)$, where $\Pi^J(q)$ and $\Pi^I(q)$ are given by $\Pi^d(q) = V_A + V_B + 2V_C - K + \mathbf{1}_{d=J}(d)S + \mu\left\{r_{P_C}^d V_C + \sum_{j\in\{A,B,C\}} r_{P_j}^d(f) V_j + q_{P_C}^d \times \sum_{j\in\{A,B,C\}} [r_{P_j}^d(s) - r_{P_j}^d(f)] V_j\right\}$. The interim and ex-post reputations in the $\Pi^J(q)$ and $\Pi^I(q)$ expressions are those associated with this equilibrium configuration. Condition $\Pi^J(q) > \Pi^I(q)$ is equivalent to $\gamma_A q_A + \gamma_B q_B > z$, where $\gamma_A = -[r_A^I(s) - r_A^I(f)](V_A + V_C) + \alpha_A^C \sum_{j\in\{A,B,C\}} [r_{P_j}^J(s) - r_{P_j}^J(f)] V_j$, $\gamma_B = \alpha_B^C \sum_{j\in\{A,B,C\}} [r_{P_j}^J(s) - r_{P_j}^J(f)] V_j$, and $z = r_A V_C + r_B V_B + r_A^I(f)[V_A + V_C] - r_{P_C}^J V_C - \sum_{j\in\{A,B,C\}} r_{P_j}^J(f) V_j - S/\mu$. Result (ii) follows from these four facts about $\gamma_A$ and $\gamma_B$: (i) $\gamma_B \ge 0$, as we consider only *sabotage-free* equilibria (indeed, if $\gamma_B < 0$, firms' joint profits would be higher when project $P_C$ failed than when it succeeded and, therefore, firms would have the incentive to make it fail); (ii) $\gamma_A$ may be positive or negative (we provide below two examples of equilibrium configurations, one for each case); (iii) $\gamma_A \ne 0$ or $\gamma_B \ne 0$, otherwise firms' optimal decision is independent of their qualities, and the equilibrium configuration under consideration is not verified; (iv) $\gamma_A < 0$ if $\gamma_B = 0$, since $r_A^I(s) > r_A^I(f)$.

For completeness, we now provide examples of each of these equilibrium configurations. In what follows, we denote consumers' initial beliefs about firm $i$'s quality by the cumulative distribution function $F_i(q_i)$ (with corresponding density $f_i(q_i)$) for all $i \in \{A,B\}$. Hence, $g(q) = f_A(q_A) \times f_B(q_B)$. We start by providing an example of the sub-type of equilibrium configuration of result (iii.a). Let $f_A(q_A) = q_A^{50}(1-q_A)^{25} / \int_0^1 \hat{q}_A^{50}(1-\hat{q}_A)^{25} d\hat{q}_A$ and $f_B(q_B) = 1$. Thus, $F_A$ is a Beta distribution and $F_B$ is a uniform distribution. This implies that $r_A = 0.6623$ and $r_B = 0.5$. Let also $\alpha_i^j = 0.5$ for all $i \in \{A,B\}$ and $j \in \{A,B,C\}$. Finally, let $V_A = V_B = 1$, $V_C = 2$, $K = 1$, and $S = 0$. Given these parameters, firms forming an alliance if and only if $\gamma_A q_A + \gamma_B q_B > z \Leftrightarrow q_B > 0.6674 - 0.6444 q_A$ characterizes the equilibrium configuration. In this equilibrium configuration, firms' interim and ex-post reputations are: $r_A^J = 0.6648$, $r_B^J = 0.6195$, $r_A^J(f) = 0.6621$, $r_A^J(s) = 0.6663$, $r_B^J(f) = 0.5528$, $r_B^J(s) = 0.6567$, $r_A^I(f) = 0.6538$, and $r_A^I(s) = 0.6667$. We now provide an example of the sub-type of equilibrium configuration of result (iii.b). Let $f_A(q_A) = 1$ and $f_B(q_B) = 1$. Hence, $F_A$ and $F_B$ are uniform distributions. This implies that $r_A = r_B = 0.5$. Let also $\alpha_A^j = 0.3$ and $\alpha_B^j = 0.7$ for all $j \in \{A,B,C\}$. Finally, let $V_A = V_B = V_C = 1$, $K = 1$, and $S = 0$. Given these parameters, firms forming an alliance if and only if $\gamma_A q_A + \gamma_B q_B > z \Leftrightarrow q_B > -0.302 + 1.07 q_A$ characterizes the equilibrium configuration. In this equilibrium configuration, firms' interim and ex-post reputations are: $r_A^J = 0.4008$, $r_B^J = 0.5928$, $r_A^J(f) = 0.3151$, $r_A^J(s) = 0.4752$, $r_B^J(f) = 0.4676$, $r_B^J(s) = 0.7016$, $r_A^I(f) = 1/3$, and $r_A^I(s) = 2/3$. ■

*Proof of Proposition 1.* As mentioned in the text, the impact of the complementarity effect on firms' joint profits is computed by setting the firms' interim and ex-post reputations equal to their initial reputations (i.e., $r_i = r_i^d = r_i^d(\varphi)$, for $i \in \{A,B\}$, $d \in \{I,J\}$, and $\varphi \in \{f,s\}$). This implies that $\Pi^J(q) = \overline{\Pi}^J = V_A + V_B + 2V_C - K + S + \mu\left\{2(\alpha_A^C r_A + \alpha_B^C r_B)V_C + \sum_{j\in\{A,B\}} (\alpha_A^j r_A + \alpha_B^j r_B) V_j\right\}$ and $\Pi^I(q) = \overline{\Pi}^I = V_A + V_B + 2V_C - K + \mu(2 r_A V_C + r_A V_A + r_B V_B)$. Therefore, $\overline{\Pi}^J - S > \overline{\Pi}^I \Leftrightarrow (r_B - r_A)(\alpha_B^A V_A - \alpha_A^B V_B + 2\alpha_B^C V_C) > 0$. ■

*Proof of Proposition 2.* Result (i) of Proposition 2 follows directly from the application of Lemma EC.1.

We now turn to the proof of result (ii). We show here that the impact of the performance of the jointly implemented project $P_C$ on the reputation of a firm may be negative. Below, in the proof of Proposition 3, we provide conditions under which this impact is positive. For a situation in which the impact is negative, consider the following example: Let $f_A(q_A) = q_A^{10}(1-q_A)^{10} / \int \hat{q}_A^{10}(1-\hat{q}_A)^{10} d\hat{q}_A$ and $f_B(q_B) = (q_B - 0.5)^{20}/$

$\int(\hat{q}_B - 0.5)^{20} d\hat{q}_B$ be the initial beliefs about the quality of firm *A* and firm *B*, respectively. Hence, $g(q) = f_A(q_A) \times f_B(q_B)$. Let also $\alpha_A^A = \alpha_A^C = 0.15$, $\alpha_A^B = 0.99$, $V_A = 5$, $V_B = 5.4315$, $V_C = 24.11$, $K = 1$, and $S = 0$. Given these parameters, in equilibrium firms form an alliance if and only if their qualities are such that $q_B > 0.0680 - 0.0882 q_A$. In this equilibrium configuration, $r_A^J(f) = 0.5280 > 0.5040 = r_A^J(s)$. ■

*Proof of Proposition 3.* Let us start with the proof of result (i), for the case of alliances where reputation-independent synergies are dominant, and therefore firms form an alliance regardless of their qualities. In this type of equilibrium configuration, as $x(q) = 1$ for all $q \in [0,1] \times [0,1]$ we have $G_1^J = G$, that is, consumers' interim beliefs are identical to their initial beliefs (see the proof of result (i) of Lemma 2 for details). Since consumers initially perceive the qualities of firms *A* and *B* as independent, they also perceive them as independent at the interim stage (after observing an alliance). This means that the covariance of $q_A$ and $q_B$ according to interim beliefs $G_1^J$ is zero (i.e., $\sigma_{A,B}^J = 0$). It follows directly from result (ii) of Lemma EC.1 that $r_i^J(s) > r_i^J > r_i^J(f)$.

We now turn to results (ii.a) and (ii.b), for the case of alliances where reputation-independent synergies are not dominant and firms' alliance formation decision depends on their qualities. We prove these results by identifying *sufficient* conditions for the impact of the performance of the jointly implemented project $P_C$ on the reputation of a firm to be positive. Turning to result (ii.a), we now show that if $\alpha_i^C$ is sufficiently high, then $r_i^J(s) > r_i^J > r_i^J(f)$ for all $i \in \{A, B\}$. By the Cauchy-Schwarz inequality, we know that $|\sigma_{A,B}^J| \leq \sqrt{\sigma_A^J \sigma_B^J}$, which implies that $\alpha_i^C \sigma_i^J + (1 - \alpha_i^C)\sigma_{A,B}^J \geq \alpha_i^C \sigma_i^J - (1 - \alpha_i^C)\sqrt{\sigma_A^J \sigma_B^J}$. Thus, for $\alpha_i^C \sigma_i^J - (1 - \alpha_i^C)\sqrt{\sigma_A^J \sigma_B^J} > 0 \Leftrightarrow \alpha_i^C > (\sqrt{\sigma_A^J \sigma_B^J}) \Big/ (\sigma_i^J + \sqrt{\sigma_A^J \sigma_B^J})$, it is necessarily the case that $\alpha_i^C \sigma_i^J + (1 - \alpha_i^C)\sigma_{A,B}^J > 0$. Hence, the result follows directly from the application of Lemma EC.1.

Finally, we prove result (ii.b) by showing that, when consumers have sufficiently low uncertainty about the quality of either firm *A* or firm *B*, the impact of the performance of project $P_C$ on the reputation of the other firm is positive. For concreteness, suppose that the uncertainty that consumers have about the quality of firm *A* is sufficiently low (the case of firm *B* is analogous); there is significant uncertainty only about firm *B*'s quality. That is, $\sigma_A^J$ is sufficiently close to zero while $\sigma_B^J$ is bounded away from zero. By the Cauchy-Schwarz inequality, we know that $|\sigma_{A,B}^J| \leq \sqrt{\sigma_A^J \sigma_B^J}$, which implies that $\sigma_{A,B}^J$ is also sufficiently close to zero. It follows from result (ii) in Lemma EC.1 that $r_B^J(s) > r_B^J > r_B^J(f)$. ■

*Proof of Proposition 4.* Let us start with the proof of result (i), for the case of alliances where reputation-independent synergies are dominant, and therefore firms form an alliance regardless of their qualities. In this type of equilibrium configuration, as $x(q) = 1$ for all $q \in [0,1] \times [0,1]$ we have $G_1^J = G$, that is, consumers' interim beliefs are identical to their initial beliefs (see the proof of result (i) of Lemma 2 for details). Hence, $r_i^J = r_i$ for $i \in \{A, B\}$.

We now turn to result (ii), for the case of alliances where reputation-independent synergies are not dominant and firms' alliance formation decision depends on their qualities. The proof consists of comparing, in equilibrium configurations where firms' decision to form an alliance depends on their qualities (i.e., in which $x(q) = 1$ if and only if $\gamma_A q_A + \gamma_B q_B > z$), firms' initial and interim reputations when they form an alliance. Let $E[\,.\,]$ denote the expectation operator according to interim beliefs $G_1^J$. By definition, $r_i^J = E[\tilde{q}_i]$ for all $i \in \{A, B\}$. By the law of iterated expectations, $r_i^J = E_{q_j}\{E[\tilde{q}_i | q_j]\}$ for all $i, j \in \{A, B\}$ and $i \neq j$. Thus, a sufficient condition for $r_i^J \geq r_i$ is that $E[\tilde{q}_i | q_j] \geq r_i$ for all $q_j$ such that $g_1^J(q) > 0$ for some $q_i$. Likewise, a sufficient condition for $r_i^J \leq r_i$ is that $E[\tilde{q}_i | q_j] \leq r_i$ for all $q_j$ such that $g_1^J(q) > 0$ for some $q_i$. Let $g_1^J(q_i | q_j) = g_1^J(q) / \int g_1^J(\hat{q}_i, q_j) d\hat{q}_i$ and let $G_1^J(q_i | q_j)$ denote its cumulative function. $G_1^J(q_i | q_j)$ is the interim conditional distribution of $\tilde{q}_i$ given $q_j$. In what follows, let $F_i(q_i)$ denote consumers' beliefs about firm $i$'s quality for all $i \in \{A, B\}$ (with corresponding density $f_i(q_i)$). Note that $g(q) = f_A(q_A) \times f_B(q_B)$. From the expression for $g_1^J(q_i | q_j)$, from $g_1^J(q) = x(q) g(q) / \int x(\hat{q}) dG(\hat{q})$, and from the fact that $g(q) = f_A(q_A) \times f_B(q_B)$, it follows that $g_1^J(q_i | q_j) = x(q) f_i(q_i) / \int x(\hat{q}_i, q_j) f_i(\hat{q}_i) d\hat{q}_i$.

Taking firm *B*, we can show that $r_B^J \geq r_B$ in any equilibrium configuration where firms' decision to form an alliance depends on their qualities. Let us fix $q_A$ such that $x(q) > 0$ for some $q_B$. If follows from result (iii) of Lemma 2 that $x(q)$ is weakly increasing in $q_B$.[1] The fact that $x(q)$ is weakly increasing in $q_B$ implies that $G_1^J(q_B|q_A)$ first-order stochastically dominates $F_B$—see $g_1^J(q_i|q_j) = x(q)f_i(q_i)/\int x(\hat{q})f_i(\hat{q}_i)d\hat{q}_i$—, which in turn means that $E[\tilde{q}_B|q_A] \geq r_B$. This implies $r_B^J \geq r_B$.

Turning to firm *A*, and following an analogous reasoning to the one used above to show that $r_B^J \geq r_B$, we can show that in the first sub-type of equilibrium configuration presented in result (iii) of Lemma 2, $r_A^J \geq r_A$. Note that, in such an equilibrium configuration, $\gamma_A \geq 0$, which implies that, given $q_B$, $x(q)$ is weakly increasing in $q_A$. Note also that an example of this sub-type of equilibrium configuration—corresponding to result (iii.a) of Lemma 2—is presented in the proof of Lemma 2.

Finally, also for firm *A*, we can show that, in the second sub-type of equilibrium configuration presented in result (iii) of Lemma 2, $r_A^J \leq r_A$. To obtain this result, consider this equilibrium configuration and fix $q_B$ such that $x(q) > 0$ for some $q_A$. In this equilibrium configuration, $\gamma_A < 0$, which implies that $x(q)$ is non-increasing in $q_A$.[2] The fact that $x(q)$ is non-increasing in $q_A$ implies that $F_A$ first-order stochastically dominates $G_1^J(q_A|q_B)$—see $g_1^J(q_i|q_j)$—, which in turn means that $E[\tilde{q}_A|q_B] \leq r_A$. This implies that $r_A^J \leq r_A$. Note also that an example of this sub-type of equilibrium configuration—corresponding to result (iii.b) of Lemma 2—is presented in the proof of Lemma 2. ■

*Proof of Proposition 5.* Let us start with the proof of result (i). Consider equilibrium configurations where firms' decision to form an alliance depends on their qualities (i.e., in which $x(q) = 1$ if and only if $\gamma_A q_A + \gamma_B q_B > z$). As shown in the proof of result (iii) of Lemma 2, $\gamma_A = -[r_A^I(s) - r_A^I(f)](V_A + V_C) + \alpha_A^C \sum_{j\in\{A,B,C\}}[r_{P_j}^J(s) - r_{P_j}^J(f)]V_j$. The proof of Proposition 4 establishes that in any such equilibrium where $\gamma_A \geq 0$—which corresponds to the sub-type of equilibrium configuration mentioned in result (iii.a) of Lemma 2—, $r_A^J \geq r_A$. Thus, the proof of result (i) of Proposition 5 consists of showing that, when $\sigma_A$ is sufficiently low (i.e., the uncertainty that consumers have about firm *A*'s quality is sufficiently low), an equilibrium configuration in which $\int x(q)g(q)dq > \varepsilon$ for a fixed $\varepsilon > 0$ (i.e., the ex-ante probability that firms form an alliance is bounded away from zero) cannot be an equilibrium configuration in which $\gamma_A < 0$—which would correspond to the sub-type of equilibrium configuration mentioned in result (iii.b) of Lemma 2. We prove this result by contradiction. Suppose that $\gamma_A < 0$. Because of $\gamma_A < 0$ and $\gamma_B \geq 0$, $G_1^J$ is such that $\sigma_{A,B}^J \geq 0$. Moreover, since $\int x(q)g(q)dq > \varepsilon$ for a fixed $\varepsilon > 0$, $G_1^J$ is non-degenerate and $\sigma_i^J$ is bounded away from zero for all $i \in \{A,B\}$. It follows that $r_i^J(s) - r_i^J(f)$ is greater than zero (from Lemma EC.1) and bounded away from zero for all $i \in \{A,B\}$. Thus, $\alpha_A^C \sum_{j\in\{A,B,C\}}[r_{P_j}^J(s) - r_{P_j}^J(f)]V_j$ is greater than zero and bounded away from zero. Next, note that for a fixed $r_A \in (0,1)$, $[r_A^I(s) - r_A^I(f)](V_A + V_C) \longrightarrow 0$ as $\sigma_A \longrightarrow 0$. This is because, as shown in Lemma EC.1, $r_A^I(s) - r_A^I(f) = \sigma_A/[r_A(1 - r_A)]$. Since $\alpha_A^C \sum_{j\in\{A,B,C\}}[r_{P_j}^J(s) - r_{P_j}^J(f)]V_j$ is greater than zero and bounded away from zero, and $[r_A^I(s) - r_A^I(f)](V_A + V_C) \longrightarrow 0$ as $\sigma_A \longrightarrow 0$, it is clear from the expression for $\gamma_A$ that, if $\sigma_A$ is sufficiently small, $\gamma_A > 0$. This is not compatible with the considered equilibrium configuration.

Result (ii) of Proposition 5 comes from the following four observations. First, $\alpha_A^C \sum_{j\in\{A,B,C\}}[r_{P_j}^J(s) - r_{P_j}^J(f)]V_j \longrightarrow 0$ as $\alpha_A^C \longrightarrow 0$. Note that $r_{P_j}^J(s) - r_{P_j}^J(f) \leq 1$ for all $j \in \{A,B,C\}$, implying that $\sum_{j\in\{A,B,C\}}[r_{P_j}^J(s) - r_{P_j}^J(f)]V_j$ is bounded. Second, as shown in Lemma EC.1, $r_A^I(s) - r_A^I(f) = \sigma_A/[r_A(1 - r_A)]$. Third, given the foregoing two observations, when $\alpha_A^C$ is sufficiently small there exists $\sigma_A^*$ such that for $\sigma_A > \sigma_A^*$, $\gamma_A < 0$.[3] Fourth, as shown in the proof of Proposition 4, in equilibrium configurations where firms' decision to form an alliance depends

[1] Note that in the characterization of the possible equilibrium configurations in result (iii) of Lemma 2, we have $\gamma_B \geq 0$. This implies that, given $q_A$ such that $x(q) > 0$ for some $q_B$, either $x(q) = 1$ for all $q_B \in [0,1]$, or $x(q) = 0$ if $q_B \leq \hat{q}_B$ for some $\hat{q}_B \in (0,1)$ and $x(q) = 1$ otherwise.

[2] Note that, given $q_B$ such that $x(q) > 0$ for some $q_A$, either $x(q) = 1$ for all $q_A \in [0,1]$, or $x(q) = 1$ if $q_A < \hat{q}_A$ for some $\hat{q}_A \in (0,1)$ and $x(q) = 0$ otherwise.

[3] Consider, for example, $\sigma_A^* = \alpha_A^C(0.25)\left(\sum_{j\in\{A,B,C\}} V_j\right)/(V_A + V_C)$.

on their qualities (i.e., in which $x(q) = 1$ if and only if $\gamma_A q_A + \gamma_B q_B > z$) and $\gamma_A < 0$, $r_A^J \leq r_A$. The fact that $\sigma_A^* \longrightarrow 0$ as $\alpha_A^C \longrightarrow 0$ follows directly from the first, second, and third observations. ■

*Proof of Proposition 6.* This result is proven by presenting a numerical example which illustrates that, in alliances where reputation-independent synergies are not dominant, ignoring the announcement and performance effects may lead to mistaken alliance formation decisions by firms.

Suppose that consumers' initial beliefs about firms' qualities are described by the following density function $g(q) = f_A(q_A) \times f_B(q_B)$, where $f_A(q_A) = q_A^{50}(1-q_A)^{25} / \int_0^1 \hat{q}_A^{50}(1-\hat{q}_A)^{25} d\hat{q}_A$ and $f_B(q_B) = 1$.[4] This implies that firms' initial reputations are $r_A = 0.6623$ and $r_B = 0.5$. Let also $\alpha_i^j = 0.5$, for $i \in \{A, B\}$ and $j \in \{A, B, C\}$, such that, in the case of an alliance, firms participate equally across all projects. Let also $V_A = V_B = 1$ and $V_C = 2$. Naturally, for reputational effects to be present, we assume some $\mu > 0$. Finally, for simplicity, we assume also that reputation-independent (cost-reducing) synergies from an alliance are non-existent (i.e., $S = 0$). Given these assumptions, firms form an alliance in equilibrium if and only if $\Pi^J(q) > \Pi^I(q) \Leftrightarrow 0.0694 q_A + 0.1081 q_B > 0.0719$. Thus, this is an equilibrium configuration where firms form an alliance if and only if their qualities are sufficiently high—as in result (iii.a) of Lemma 2. Moreover, in equilibrium firms' interim and ex-post reputations are endogenously determined and correspond to: $r_A^J = 0.6648$, $r_B^J = 0.6195$, $r_A^J(f) = 0.6621$, $r_A^J(s) = 0.6663$, $r_B^J(f) = 0.5528$, $r_B^J(s) = 0.6567$, $r_A^I = r_A = 0.6623$, $r_B^I = r_B = 0.5$, $r_A^I(f) = 0.6538$, $r_A^I(s) = 0.6667$, and $r_B^I(f) = r_B^I(s) = r_B = 0.5$.

Taking the above equilibrium values, we now decompose the impact of the different reputational effects on joint profits, and thus on firms' equilibrium decision. The procedure used in this decomposition is fully described in Figure EC.1.

We start by taking the perspective of firms that would only consider the complementarity effect and potential reputation-independent synergies, thus ignoring the performance and announcement effects. This corresponds to assuming that firms' reputations remained equal to their initial reputations regardless of firms' decision and project outcomes. Thus, when deciding whether to form an alliance, firms would only consider $\overline{\Pi}^J$ and $\overline{\Pi}^I$, which correspond to joint profits if firms' reputations remained the same as their initial reputations. In particular, firms would decide to form an alliance if and only if $\overline{\Pi}^J > \overline{\Pi}^I \Leftrightarrow \overline{\Pi}^J - \overline{\Pi}^I > 0 \Leftrightarrow \Delta\Pi_{Comp.} + S > 0$, where $\Delta\Pi_{Comp.} = \overline{\Pi}^J - S - \overline{\Pi}^I$ corresponds to the profit impact of the complementary effect. In our numerical example, since we assume no reputation-independent synergies from an alliance (i.e., $S = 0$), we have $\Delta\Pi_{Comp.} + S = -0.3246 + 0 < 0$, meaning that in this case firms would choose independent project implementation regardless of their qualities. As discussed before, if and only if firms' combined qualities are sufficiently high, an alliance is the optimal choice. Thus, considering only the complementarity effect (and potential reputation-independent synergies) may lead to a mistaken decision, in the form of independent project implementation in situations where an alliance is the optimal choice.

We now consider the impact of the announcement effect on firms' decision. For a given decision $d \in \{I, J\}$, the profit impact of the announcement effect can be computed as $\Delta\Pi_{Anno.}^d = \overline{\overline{\Pi}}^d - \overline{\Pi}^d$, where $\overline{\overline{\Pi}}^d$ corresponds to joint profits if firms' reputations remained equal to their interim reputations. In our numerical example, $\Delta\Pi_{Anno.}^J = 0.366$ and $\Delta\Pi_{Anno.}^I = 0$, meaning that the profit impact of the announcement effect is positive if firms form an alliance, and zero under independent project implementation.[5] Thus, the impact of the announcement effect on firms' incentive to form an alliance (given by $\Delta\Pi_{Anno.}^J - \Delta\Pi_{Anno.}^I = 0.366$) is positive. If firms considered the complementarity and announcement effects and potential reputation-independent synergies, but ignored the performance effect, they would decide to form an alliance if and only if $\overline{\overline{\Pi}}^J > \overline{\overline{\Pi}}^I \Leftrightarrow \overline{\Pi}^J + \Delta\Pi_{Anno.}^J > \overline{\Pi}^I + \Delta\Pi_{Anno.}^I \Leftrightarrow \Delta\Pi_{Comp.} + S + \left[\Delta\Pi_{Anno.}^J - \Delta\Pi_{Anno.}^I\right] > 0$. In our numerical example, we have $\Delta\Pi_{Comp.} + S + \left[\Delta\Pi_{Anno.}^J - \Delta\Pi_{Anno.}^I\right] = -0.3246 + 0 + [0.366 - 0] = 0.0414 > 0$, meaning that the joint impact of the complementarity and announcement effects on firms' incentive to form an alliance is positive. As a result, in this case firms would choose to form an alliance regardless of their qualities. Since, as discussed before, the optimal decision is an alliance if and only if firms' combined qualities are sufficiently high, ignoring the performance effect may lead to a mistaken decision, in the form of an alliance in situations where independent project implementation is the optimal choice.

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[4] The fact that qualities $q_A$ and $q_B$ enter separately $g(q)$ through the marginal density functions $f_A(q_A)$ and $f_B(q_B)$ means that qualities $q_A$ and $q_B$ are initially perceived by consumers as independent. Furthermore, given the specific marginal density functions considered, the associated cumulative distribution functions $F_A(q_A)$ and $F_B(q_B)$ are, respectively, a Beta distribution and a uniform distribution.

[5] Note that $\overline{\overline{\Pi}}^I = \overline{\Pi}^I$, since the interim reputations are the same as the initial reputations under independent project implementation.

1. Firms form an alliance in equilibrium if and only if:

$$\Pi^J(q) > \Pi^I(q) \Leftrightarrow \Pi^J(q) - \Pi^I(q) > 0$$

2. For decision $d \in \{I, J\}$, consider the following:
   - $\overline{\Pi}^d$: Firms' joint profits if firms' reputations remained equal to their initial reputations
   - $\overline{\overline{\Pi}}^d$: Firms' joint profits if firms' reputations remained equal to their interim reputations

3. Adding and subtracting $\overline{\Pi}^J$, $\overline{\Pi}^I$, $\overline{\overline{\Pi}}^J$, $\overline{\overline{\Pi}}^I$, and $S(\geq 0)$ from the above expression, we get:

$$\underbrace{\left(\Pi^J(q) - \overline{\overline{\Pi}}^J\right)} - \underbrace{\left(\Pi^I(q) - \overline{\overline{\Pi}}^I\right)} + \underbrace{\left(\overline{\overline{\Pi}}^J - \overline{\Pi}^J\right)} - \underbrace{\left(\overline{\overline{\Pi}}^I - \overline{\Pi}^I\right)} + \underbrace{(\overline{\Pi}^J - S - \overline{\Pi}^I)} + S > 0$$

$$\Leftrightarrow \Delta\Pi^J_{Perf.}(q) - \Delta\Pi^I_{Perf.}(q) + \Delta\Pi^J_{Anno.} - \Delta\Pi^I_{Anno.} + \Delta\Pi_{Comp.} + S > 0$$

   where:
   - $S$: Reputation-independent synergies from an alliance
   - $\Delta\Pi_{Comp.}$: Profit impact of the complementarity effect (i.e., assuming firms' reputations remained equal to their initial reputations)
   - $\Delta\Pi^d_{Anno.}$, for $d \in \{I, J\}$: Profit impact of the announcement effect relative to the case in which firms' reputations remained equal to their initial reputations
   - $\Delta\Pi^d_{Perf.}(q)$, for $d \in \{I, J\}$: Profit impact of the performance effect relative to the case in which firms' reputations remained equal to their interim reputations

4. Aggregating $\Delta\Pi^J_{Perf.}(q) - \Delta\Pi^I_{Perf.}(q)$ and $\Delta\Pi^J_{Anno.} - \Delta\Pi^I_{Anno.}$ above, we have:

$$\underbrace{\left[\Delta\Pi^J_{Perf.}(q) - \Delta\Pi^I_{Perf.}(q)\right]} + \underbrace{\left[\Delta\Pi^J_{Anno.} - \Delta\Pi^I_{Anno.}\right]} + \underbrace{\Delta\Pi_{Comp.}} + \underbrace{S} > 0$$

| | | | |
|---|---|---|---|
| Impact of the performance effect on firms' incentive to form an alliance | Impact of the announcement effect on firms' incentive to form an alliance | Impact of the complementarity effect on firms' incentive to form an alliance | Impact of reputation-independent synergies on firms' incentive to form an alliance |

Figure EC.1 Decomposition of the Impact of the Three Reputational Effects on Firms' Equilibrium Decision

Finally, we turn to the performance effect. For a given decision $d \in \{I, J\}$, the profit impact of the performance effect can be computed as $\Delta\Pi^d_{Perf.}(q) = \Pi^d(q) - \overline{\overline{\Pi}}^d$.[6] In our numerical example, $\Delta\Pi^J_{Perf.}(q) = 0.1081(q_A + q_B) - 0.1388$ and $\Delta\Pi^I_{Perf.}(q) = 0.0387q_A - 0.0255$, meaning that, if firms' qualities are sufficiently high, the profit impact of the performance effect is positive regardless of their decision. The impact of the performance effect on firms' incentive to form an alliance is given by $\Delta\Pi^J_{Perf.}(q) - \Delta\Pi^I_{Perf.}(q) = 0.0694q_A + 0.1081q_B - 0.1133$, which is increasing in both quality $q_A$ and quality $q_B$.[7] If firms considered all three reputational effects and potential reputation-independent synergies, they would choose an alliance if and only if $\Pi^J(q) > \Pi^I(q) \Leftrightarrow \overline{\Pi}^J + \Delta\Pi^J_{Anno.} + \Delta\Pi^J_{Perf.}(q) > \overline{\Pi}^I + \Delta\Pi^I_{Anno.} + \Delta\Pi^I_{Perf.}(q) \Leftrightarrow \Delta\Pi_{Comp.} + S + \left[\Delta\Pi^J_{Anno.} - \Delta\Pi^I_{Anno.}\right] +$

[6] Note that, unlike $\Delta\Pi_{Comp.}$ and $\Delta\Pi^d_{Anno.}$, $\Delta\Pi^d_{Perf.}(q)$ depends *directly* on firms' qualities $q$, as firms' qualities determine the likelihood of success (i.e., the performance) of project $P_C$.

[7] This justifies why, in the equilibrium configuration of this numerical example, the likelihood of an alliance is increasing in firms' qualities.

$\left[\Delta\Pi^{J}_{Perf.}(q) - \Delta\Pi^{I}_{Perf.}(q)\right] > 0$. In our numerical example, we have $\Delta\Pi_{Comp.} + S + \left[\Delta\Pi^{J}_{Anno.} - \Delta\Pi^{I}_{Anno.}\right] + \left[\Delta\Pi^{J}_{Perf.}(q) - \Delta\Pi^{I}_{Perf.}(q)\right] = -0.3246 + 0 + [0.366 - 0] + [0.0694q_A + 0.1081q_B - 0.1133] = 0.0694q_A + 0.1081q_B - 0.0719$, which corresponds to the equilibrium configuration that was described at the beginning.

This numerical example underscores the idea that, when reputational considerations are important determinants of firms' decision to form an alliance, firm managers should consider the different reputational effects in that decision, as not doing so may lead to suboptimal choices. In this example, while for high qualities of the two firms the gains from an alliance stemming from the performance and announcement effects dominate the losses stemming from the complementary effect; for low qualities of the two firms the losses from an alliance stemming from the performance and complementarity effects dominate the gains stemming from the announcement effect. Thus, high-quality firms would wrongly implement their projects independently if their managers only considered the complementarity effect. ■

## EC.3 Model Extension: Alliance scope

Despite their practical relevance, alliance scope decisions have been somewhat understudied in the literature. Existing research largely focuses on learning alliances (i.e., alliances where partners' main objective is to learn from each other), in which there is a latent tension between firms' incentives to exchange knowledge to achieve the alliance's objectives (i.e., common benefits) and their incentives to learn as much (and as fast) as possible to derive advantages outside the context of the alliance (i.e., private benefits) (Hamel et al. 1989, Hamel 1991). The relative magnitude of these common and private benefits is determined by the interplay between the scope of an alliance and the extent of the outside activities of alliance partners. As such, the relative scope of an alliance is pointed out as a determinant of alliance partners' commitment to it (e.g., Khanna, 1998, Khanna et al. 1998, Baum et al. 2000, Oxley and Sampson 2004, Lunnan and Haugland 2008).

In our model, the assumption that firms' objective is to maximize their joint profits—and hence that surplus can be transferred between firms without frictions—blurs the above distinction between common and private benefits. This is attested by the fact that our model allows for profits stemming from any projects (within or outside an alliance) to be shared between alliance partners. Thus, instead of addressing potential tensions between common and private benefits in alliances, the analysis presented here is on how reputational considerations affect firms' joint profits and, from that standpoint, firms' optimal choice of alliance scope.

An analytically tractable way of examining firms' choice of alliance scope is to compare two contrasting alliance options: a narrow alliance ($d = N$), where firms jointly implement only project $P_C$ and implement their other projects (projects $P_A$ and $P_B$) independently, and a wide alliance ($d = W$), where firms jointly implement all their projects (projects $P_A$, $P_B$, and $P_C$). While the assumption that firms jointly implement all their projects in a wide alliance may seem extreme, it establishes a clear contrast to a narrow alliance, and thus a convenient benchmark.

To analyze how reputational considerations affect firms' choice of alliance scope, we make some additional assumptions. To isolate alliance scope (captured by the number of jointly implemented projects) from other dimensions of firms' collaboration, we consider that firms *A* and *B* have the same participation levels in project $P_C$ under a narrow alliance and under a wide alliance. This implies that the quality of project $P_C$ is the same in both cases ($q^N_{P_C} = q^w_{P_C} = \alpha^C_A q_A + \alpha^C_B q_B$).[8] Thus, the difference between the two types of alliances is that in a narrow alliance we have $\alpha^A_A = \alpha^B_B = 1$ and $\alpha^B_A = \alpha^A_B = 0$, whereas in a wide alliance we have $\alpha^j_i \in (0,1)$ for $i, j \in \{A, B\}$. Moreover, we assume that reputation-independent synergies are the same in the case of a wide alliance and in the case of a narrow alliance and, as before, given by $S$.[9] Finally, to focus solely on the choice between a wide alliance and a narrow alliance, we consider that reputation-independent synergies are sufficiently strong, so that it is always optimal for firms to choose one of the two types of alliances over independent project implementation. The formal condition for this to happen is $S > \mu[r_A V_C + r_B V_B + r^I_A(s)(V_A + V_C)]$. Note that this condition is the same as the one presented in section 3 of the paper, to ensure that reputation-independent synergies were dominant.

Since reputation-independent synergies are assumed to be the same under a wide alliance and under a narrow alliance, the choice between the two is determined by reputational considerations. It turns out that these reputational considerations are captured by the cumulative impact of the three reputational effects that we previously identified in our main analysis: the complementarity effect, the performance effect, and the announcement effect.

[8] It is important to note that, while this assumption considerably simplifies the analysis, it does not affect the main results.

[9] Presumably, reputation-independent synergies should differ depending on firms' alliance scope. Nonetheless, it is difficult to establish *a priori* which type of alliance will have the strongest reputation-independent synergies. For example, while a wide alliance may provide more opportunities for resource sharing and rationalization of activities, it may also be associated with more complex and costly coordination processes.

As before, we isolate the impact of the complementarity effect on firms' joint profits by setting firms' interim and ex-post reputations equal to their initial reputations (i.e., $r_i = r_i^d = r_i^d(\varphi)$, for $i \in \{A, B\}$, $d \in \{N, W\}$, and $\varphi \in \{f, s\}$). This implies that firms' joint profits become invariant to their qualities (i.e., $\Pi^d(q) = \overline{\Pi}^d$, for $d \in \{N, W\}$).

PROPOSITION EC.1. *A wide alliance has a greater impact than a narrow alliance on firms' joint profits through the complementarity effect if and only if the wider alliance scope contributes to combine the high reputation of a given firm with the high-value project(s) of the other firm (i.e., if and only if* $\overline{\Pi}^W - S > \overline{\Pi}^N - S \Leftrightarrow (r_B - r_A)(\alpha_B^A V_A - \alpha_A^B V_B) > 0$*).*

*Proof of Proposition EC.1.* We first introduce some additional notation that is necessary to write firms' expected joint profits. If firms decide to form a wide alliance, let $r_{P_C}^W$ and $r_{P_j}^W(\varphi)$ denote, respectively, the reputation of project $P_C$ in period one and the reputation of project $P_j$ (for $j \in \{A, B, C\}$ and $\varphi \in \{f, s\}$) in period two. Likewise, if firms decide to form a narrow alliance, let $r_{P_C}^N$ and $r_{P_j}^N(\varphi)$ denote, respectively, the reputation of project $P_C$ in period one and the reputation of project $P_j$ (for $j \in \{A, B, C\}$ and $\varphi \in \{f, s\}$) in period two. Firms' expected joint profits depend on $d \in \{N, W\}$, and are denoted by $\Pi^d(q) = V_A + V_B + 2V_C - K + S + \mu\left\{r_{P_C}^d V_C + \sum_{j\in\{A,B,C\}} r_{P_j}^d(f)V_j + q_{P_C}^d \times \sum_{j\in\{A,B,C\}}[r_{P_j}^d(s) - r_{P_j}^d(f)]V_j\right\}$, where $r_{P_C}^d = \alpha_A^C r_A^d + \alpha_B^C r_B^d$ and $r_{P_C}^d(\varphi) = \alpha_A^C r_A^d(\varphi) + \alpha_B^C r_B^d(\varphi)$ (for $d \in \{N, W\}$ and $\varphi \in \{f, s\}$). Furthermore, we assume that $q_{P_C}^N = q_{P_C}^W = \alpha_A^C q_A + \alpha_B^C q_B$. Since under a wide alliance projects $P_A$ and $P_B$ are assumed to be implemented jointly, whereas under a narrow alliance they are assumed to be implemented independently, respectively, by firm *A* and by firm *B*, we have $r_{P_j}^W(\varphi) = \alpha_A^j r_A^W(\varphi) + \alpha_B^j r_B^W(\varphi)$ and $r_{P_j}^N(\varphi) = r_j^N(\varphi)$, for $j \in \{A, B\}$.

The proof of Proposition EC.1 is analogous to that of Proposition 1. If we set $r_i = r_i^d = r_i^d(\varphi)$ for all $i \in \{A, B\}$, $d \in \{N, W\}$, and $\varphi \in \{f, s\}$, then $\Pi^W(q) = \overline{\Pi}^W = V_A + V_B + 2V_C - K + S + \mu\{2(\alpha_A^C r_A + \alpha_B^C r_B)V_C + (\alpha_A^A r_A + \alpha_B^A r_B)V_A + (\alpha_A^B r_A + \alpha_B^B r_B)V_B\}$ and $\Pi^N(q) = \overline{\Pi}^N = V_A + V_B + 2V_C - K + S + \mu\{2(\alpha_A^C r_A + \alpha_B^C r_B)V_C + r_A V_A + r_B V_B\}$. Therefore, $\overline{\Pi}^W - S > \overline{\Pi}^N - S \Leftrightarrow (r_B - r_A)(\alpha_B^A V_A - \alpha_A^B V_B) > 0$. ■

The condition in Proposition EC.1 highlights that, from the standpoint of the profit impact of the complementarity effect, a wide alliance is preferred to a narrow alliance if one firm has a high reputation and the other firm has high-value projects. For example, if $r_B > r_A$, the condition is satisfied as long as $V_A$ is sufficiently large relative to $V_B$. More generally, a greater potential to combine the high reputation of one firm with the high-value projects of the other firm, in and of itself, promotes a greater alliance scope.

Beyond the complementarity effect, the optimal choice of alliance scope is also influenced by the announcement and performance effects. As before, the analysis of these two effects is based on firms' interim and ex-post reputations, which depend on the verified equilibrium configuration. Thus, the analysis of the announcement and performance effects requires the characterization of firms' equilibrium strategies. This is done in Lemma EC.2.

LEMMA EC.2. *Two possible types of equilibrium configurations exist where either a wide alliance or a narrow alliance can be optimal:*

*(i) Firms form the wide alliance if and only if the quality of the joint project* $P_C$ *is above a given threshold (i.e.,* $\Pi^W(q) > \Pi^N(q) \Leftrightarrow \alpha_A^C q_A + \alpha_B^C q_B > y$*, where* $y$ *is a scalar), and form the narrow alliance otherwise.*

*(ii) Firms form the narrow alliance if and only if the quality of the joint project* $P_C$ *is above a given threshold (i.e.,* $\Pi^N(q) > \Pi^W(q) \Leftrightarrow \alpha_A^C q_A + \alpha_B^C q_B > y$*, where* $y$ *is a scalar), and form the wide alliance otherwise.*

*Proof of Lemma EC.2.* Recall that we are considering situations where independent project implementation is always dominated and, therefore, in equilibrium firms either form a narrow alliance or a wide alliance—that is, $x(q) \in \{0,1\}$ for all $q \in [0,1] \times [0,1]$, where in this case $x(q) = 0$ if firms choose to form a narrow alliance (i.e., $d = N$), and $x(q) = 1$ if firms choose to form a wide alliance (i.e., $d = W$). In these equilibrium configurations, firms form a wide alliance if and only if $\Pi^W(q) > \Pi^N(q)$, and form a narrow alliance otherwise. Using the expressions for firms' expected joint profits and the fact that $q_{P_C}^N = q_{P_C}^W = \alpha_A^C q_A + \alpha_B^C q_B$, we have $\Pi^W(q) > \Pi^N(q) \Leftrightarrow (\alpha_A^C q_A + \alpha_B^C q_B) \times \left\{\sum_{j\in\{A,B,C\}}\left[r_{P_j}^W(s) - r_{P_j}^W(f)\right]V_j - \sum_{j\in\{A,B,C\}}\left[r_{P_j}^N(s) - r_{P_j}^N(f)\right]V_j\right\} > (r_{P_C}^N - r_{P_C}^W)V_C +$

$\sum_{j\in\{A,B,C\}}\left(r_{P_j}^N(f)-r_{P_j}^W(f)\right)V_j$. The term inside the squiggly brackets represents the difference between the derivative of firms' expected joint profits with respect to the quality of project $P_C$ under a wide alliance and the equivalent derivative under a narrow alliance (i.e., $\left(\partial\Pi^W(q)/\partial q_{P_C}^W\right)-\left(\partial\Pi^N(q)/\partial q_{P_C}^N\right)$). If the term inside the squiggly brackets is positive, we obtain the equilibrium configuration described in result (i) of Lemma EC.2. If the term inside the squiggly brackets term is negative, we obtain the equilibrium configuration described in result (ii) of Lemma EC.2. If the term inside the squiggly brackets is zero, firms either form a narrow alliance or a wide alliance regardless of their qualities. If the term inside the squiggly brackets is not zero, the relevant condition for each of the two equilibrium configurations in results (i) and (ii) of Lemma EC.2 can be written as $\alpha_A^C q_A+\alpha_B^C q_B>y$, where $y$ aggregates all the other terms of the relevant inequality. ■

The two equilibrium configurations identified in Lemma EC.2 are represented, respectively, in Panel A and Panel B of Figure EC.2. In both cases, downward sloping lines in the space of firm qualities $(q_A, q_B)$ separate the region where firms form a wide alliance from the region where firms form a narrow alliance.

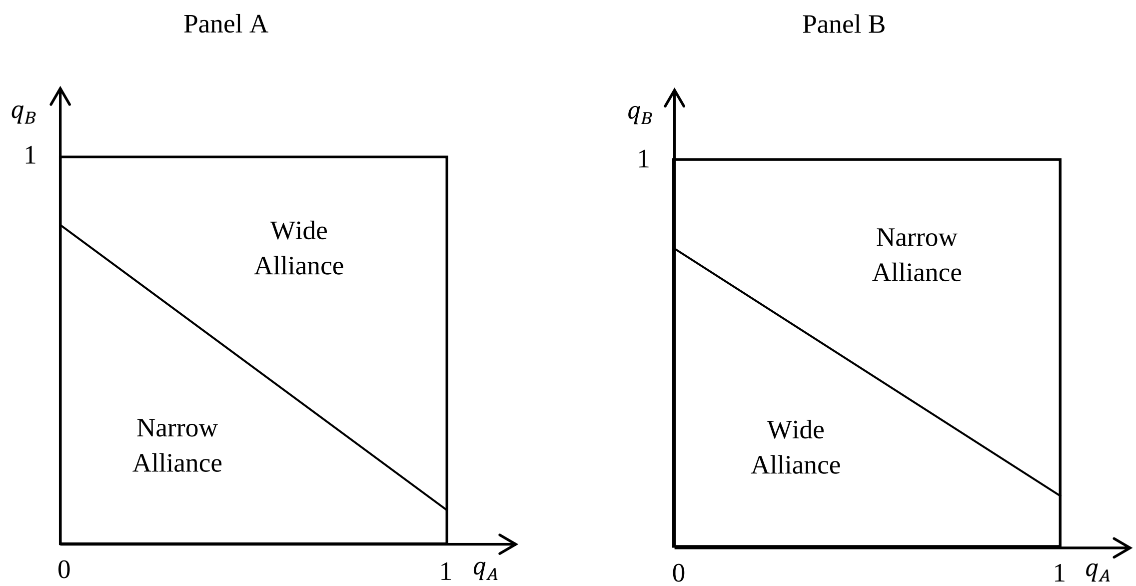


Figure EC.2 Equilibrium Configurations where a Wide and a Narrow Alliance Occur for Different Firm Qualities

In the equilibrium configuration depicted in Panel A, firms form a wide alliance when their qualities are high and form a narrow alliance when their qualities are low. This means that the impact of firms' qualities on joint profits through a success of (joint) project $P_C$ in period one—that is, the impact of the performance effect on firms' joint profits—is higher when the scope of their collaboration goes beyond project $P_C$. The opposite happens in Panel B. Moreover, since in Panel A firms are more likely to form a wide alliance if their qualities are high, the announcement of a wide (narrow) alliance signals a high (low) quality of both firms to consumers. In Panel B, the opposite happens.

As in our main analysis, the announcement and performance effects may be consequential for firms' choice of alliance scope, since in equilibrium these two effects may counter and dominate the complementarity effect. Proposition EC.2 states this result formally.

> PROPOSITION EC.2. *The combined impact of the announcement and performance effects on firms' joint profits under a wide alliance (versus a narrow alliance) may counter and dominate the impact of the complementarity effect, thereby determining firms' optimal choice of alliance scope.*

*Proof of Proposition EC.2.* This result is proven by presenting a numerical example which, in the same spirit of the prior numerical example that was used to prove Proposition 6, illustrates how ignoring the announcement and performance effects may lead to mistaken decisions between a narrow alliance and a wide alliance. Suppose that consumers' initial beliefs about firms' qualities are described by the density function $g(q) = f_A(q_A) \times f_B(q_B)$, where $f_A(q_A) = (1 - q_A)^{10} / \int_0^1 (1 - \hat{q}_A)^{10} d\hat{q}_A$ and $f_B(q_B) = q_B^8 (1 - q_B)^4 / \int_0^1 \hat{q}_B^8 (1 - \hat{q}_B)^4 d\hat{q}_B$. This implies that firms' initial reputations are $r_A = \int_0^1 \hat{q}_A f_A(\hat{q}_A) d\hat{q}_A \approx 0.083333$ and $r_B = \int_0^1 \hat{q}_B f_B(\hat{q}_B) d\hat{q}_B = 9/14 = 0.64286$. Let also $\alpha_A^A = \alpha_A^C = 0.5$, $\alpha_A^B = 0.95$, $V_A = 10$, $V_B = 35$, and $V_C = 35$. Note that the values of $S$, $K$, and $\mu$ are irrelevant as long as $S$ is sufficiently large relative to $\mu$, so that a narrow alliance and a wide alliance are always better than independent project implementation. Given these assumptions, firms form a wide alliance in equilibrium if and only if $\Pi^W(q) > \Pi^N(q) \Leftrightarrow 0.5q_A + 0.5q_B > 0.299$, and form a narrow alliance otherwise.

We now compare the optimal decision above with the decision that would be made by firms that would only consider the complementarity effect and potential reputation-independent synergies, thus ignoring the performance and announcement effects. This corresponds to assuming that firms' reputations remained equal to their initial reputations regardless of firms' decision and project outcomes. Thus, when deciding whether to form a narrow alliance or a wide alliance, firms would only consider $\overline{\Pi}^W$ and $\overline{\Pi}^N$, which correspond to joint profits if firms' reputations remained the same as their initial reputations. In particular, firms would decide to form a wide alliance if and only if $\overline{\Pi}^W > \overline{\Pi}^N$. In our numerical example, we have $\overline{\Pi}^W - \overline{\Pi}^N = \mu 32.944 - \mu 48.75 = -\mu 15.806 < 0$, meaning that in this case firms would choose to form a narrow alliance regardless of their qualities. As discussed before, if firms' qualities are sufficiently high, a wide alliance is the optimal choice. Thus, considering only the complementarity effect (and potential reputation-independent synergies) may lead to a mistaken decision, in the form of a narrow alliance in situations where a wide alliance is the optimal choice.

This numerical example corresponds to a situation where the combined impact of the announcement and performance effects on the optimal choice between a narrow alliance and a wide alliance may counter and dominate the impact of the complementarity effect. Akin to the previous numerical example in the proof of Proposition 6, this example emphasizes that firm managers should consider the different reputational effects, as not doing so may lead to suboptimal alliance scope choices. ■

Overall, the analysis presented in this model extension highlights the potential importance of the three reputational effects for firms' choice of alliance scope.

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